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ABUNDANCES IN THE RED GIANTS OF M13 AND M22

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ABSTRACT

Abundances of Ca, Na, and Fe are derived for 10 red giant branch stars in the globular clusters M13 and M22, by means of model atmosphere analyses of high-dispersion spectra. It was found that M13 and M22 represent two different types of globular clusters. M13 is similar to M92 and many other globular clusters in that it displays variations in CH and CN band strengths, while showing no variation in either Ca or Fe abundance. The Na abundance was found to vary and its variation correlates with CN band strengths. This suggests that the source of the observed CN band strength variation is the same as that of the Na variation. M22, on the other hand, is similar to ω Cen, in that it displays variations in Ca, Na, and Fe abundances (Δ[Fe/H] $\sim \Delta$[Ca/H] ~ 0.4) and these variations correlate with variations in CH and CN band strengths. These correlations suggest that the cause of the variations in M22 is largely "primordial" and mixing may only make a relatively small contribution to the C and N variations observed.

Subject headings: clusters: globular — stars: abundances

1. INTRODUCTION

There have been many studies indicating that the giants of globular clusters differ in their chemical composition. The observation of C^{12}, C^{13}, and nitrogen (and possibly oxygen) abundance variations seems well established. The case for abundance variations in elements heavier than these seems less clear, except for ω Centauri.

Stars in ω Cen have a large variation in the abundances of elements other than carbon and nitrogen, e.g., the calcium variations found by Freeman & Rodgers (1975). Stars in M22 have also been found to vary in CN, CH, and Ca H and K line strengths (Norris & Freeman 1983), as well as Na and Al resonance line strengths. Norris & Freeman attributed these variations to an abundance spread within M22, although this conclusion is controversial. Cohen (1981), using high-resolution spectroscopic data, found only a variation in Na abundance. Stars in M13 have been found to have variations in CH and CN band strengths (Suntzeff 1980, 1981), but do not have Fe or Ca variations, although Na and Al variations have been found (Cohen 1981; Wallerstein, Leep, & Oke 1987).

The goal of this paper is to attempt to determine the Ca, Fe, and Na abundances of M22 and M13 red giants and correlate these abundances with studies indicating variations in CH and CN band strengths. The M13 study serves as a foil to the M22 one. Fe and Ca were chosen because the spectra of giants contain many lines of these elements and neither is thought to be produced in the interiors of low-mass giants. Na was chosen in an attempt to confirm its observed variation in both clusters. If it is found that there are Ca, Na, and Fe abundance variations and these variations correlate with CN and CH band strengths, then this implies that the inhomogeneity in C and N is partly due to "primordial" enrichment and not completely due to the transfer of processed material to the stellar surface. If, however, there is no variation in Ca or Fe, or their variation does not correlate with the CN and CH band strengths, then the cause of the inhomogeneity is probably internal.

In order to measure small variations in abundances, one would like to observe a region of the spectrum that contains lines that are both relatively strong, so that accurate equivalent widths can be measured, and of course, abundance-sensitive. A logical region of the spectrum to observe is the near-infrared.

The strongest lines in the IR are those of the Ca triplet. It is desirable to understand their behavior and judge how useful they are as a temperature, gravity, or abundance indicator. Fortunately, there have been several studies measuring the behavior of the Ca triplet as a function of these parameters. Some indicated that the Ca triplet is most sensitive to surface gravity (Cohen 1978a; Jones, Allion, & Jones 1984; Carter, Visvanathan, & Pickles 1986), becoming stronger with decreasing gravity. However, these studies investigated the properties of the Ca triplet for metal-rich stars ([Fe/H] > -0.3). Recently, Diaz, Terlevich, & Terlevich (1989) investigated stars with a range of metallicities. They found that for stars with [Fe/H] < -0.3, the equivalent width of the Ca triplet is most sensitive to abundance.

Da Costa (1986) presents a plot of absolute I magnitude, M_I, versus the sum of the equivalent width of the Ca lines at 8542 Å and 8662 Å, $\Sigma(W)$, for giants of a few globular clusters with differing metallicities (see Fig. 1). The relationship between M_I and $\Sigma(W)$ for giants of uniform abundance is approximately linear and the giant branches of clusters with different metallicities are well separated. This method, while implying that the Ca triplet is sensitive to abundance in stars of low metallicity, also provides a very powerful way to deduce abundances and demonstrate the existence of abundance variations.

With this in mind, we attempted to analyze the abundance of M13 and M22 stars using the Ca triplet in addition to other weaker lines.

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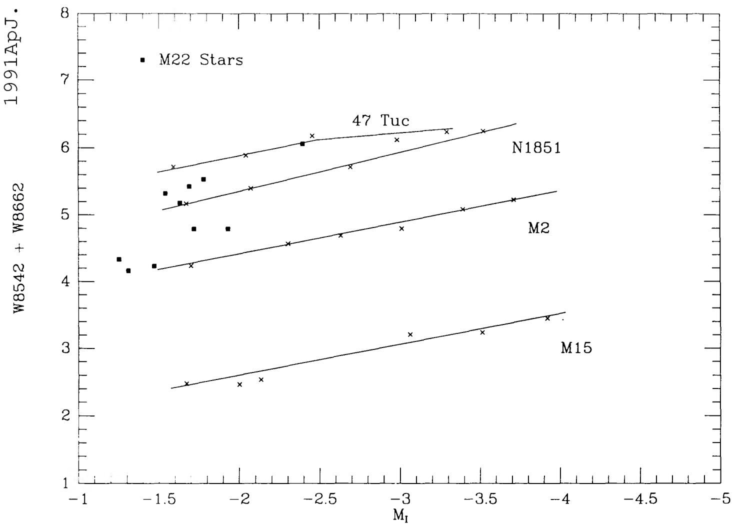

FIG. 1.—The figure shows the sum of the equivalent widths of the lines 8542 and 8662 of the Ca triplet plotted vs. M_I. The crosses represent data taken from DaCosta (1986), and the lines, his linear fits to the data. The squares represent our data for M22. The scatter seems to indicate that M22 is not uniform in Ca abundance.

2. OBSERVATIONAL MATERIAL

The spectra were taken with the Palomar 5 m using the double-spectrograph (Oke & Gunn 1982) equipped with two TI CCDs. The red side spectra had a range in wavelength of about 8200–8800 Å with about 0.8 Å pixel^{-1}. The blue side spectra were from 5560 to 5980 Å with about 0.5 Å pixel^{-1}. Each spectra had a resolution of several pixels. The signal-to-noise ratio for all spectra was greater than about 30.

Equivalent widths were measured by fitting a Lorentzian profile to the Ca triplet lines and Gaussian to all other lines. Estimates of continuum level were made by eye and all profile fits were checked by eye. Deciding on the continuum level for the red spectra was relatively straightforward due to the large wavelength coverage of the spectra and to the lack of line absorption in this spectral region. The line blanketing is much greater for the blue spectra and, as a result, the continuum level is much less certain.

The absolute magnitudes and unreddened $B-V$ colors of the stars were taken from the *UBV* photometry of Cathey (1974) for M13 and Evans (1975) for M22. This was combined with color excesses and unreddened distance moduli to determine the absolute magnitudes and unreddened colors. Bell & Dickens (1980) gave the color excess [$E(B-V) = 0.02$] and distance modulus (14.3) used for M13, while the color excess [$E(B-V) = 0.42$] for M22 was taken from Crocker (1989), and the distance modulus (12.20) from Cudworth & Peterson (1987). These absolute magnitudes and unreddened colors were then plotted on an interpolation of the theoretical temperature-gravity grids of Bell & Gustafsson (1978) to determine the effective temperatures and gravities of the stars, assuming a general metal abundance of [A/H] = -1.5 and mass of 0.8 $M_\odot$. The resulting temperatures and gravities are shown in Tables 1 and 2.

In addition, all 10 stars in M22 used in this study have observed K magnitudes (I-80 and IV-20 by Frogel, Persson, & Cohen 1983; the remaining eight are unpublished). We determined an additional set of temperatures by plotting unreddened $V-K$ colors against their absolute magnitudes of an interpolation of the theoretical temperature-gravity grid of Bell & Gustafsson (1989), again assuming [A/H] = -1.5 and a mass of 0.8 $M_\odot$ for each star. We found a tight correlation between the $B-V$ and $V-K$ derived temperatures with a random error of about 100 K. Also, Cohen et al. (1978) give a temperature for the star I-18 in M13 derived from $V-K$ (unfortunately, this was the only star for which they derived a temperature in our sample). Our $B-V$ derived temperature agrees well with their temperature for this star.

TABLE 1

DATA USED TO DERIVE $T_{\rm eff}$ AND log g FOR M22 STARS

M22 Star	$E(B-V) = 0.42$		$V_0 - M_V = 12.20$	
	V_0	$(B-V)_0$	$T_{\rm eff}$	log g
I-116	11.56	0.78	4950	2.00
I-108	11.49	0.69	5100	2.00
I-53	11.38	0.91	4700	1.75
III-25	11.36	0.75	5000	1.90
III-35	11.05	0.93	4700	1.60
IV-20	10.79	1.07	4500	1.25
IV-24	11.31	0.94	4650	1.70
I-80	11.25	0.94	4650	1.65
I-85	11.18	0.80	4900	1.80
I-27	11.08	0.79	4950	1.75

NOTE.—*UBV* photometry from Evans 1975.

The determination of cluster membership is an important component of any analysis seeking to demonstrate the existence of chemical inhomogeneities. Cudworth & Monet (1979) give the probability of membership for M13 stars, suggesting that all of the stars used in this analysis have 99% probabilities of being members. The same technique was performed by Cudworth (1986) for M22. His results suggest that eight of the stars used have at least a 90% probability of membership. Of the other two, III-25 was given only a 20% probability of being a member and a membership probability was not given for I-108. However, no radial velocity abnormality was noted for either star and their removal from the data set will have little effect on the conclusions of this study.

3. REDUCTION AND ANALYSIS

3.1. *Atmosphere Calculations*

Model atmospheres for this analysis were constructed using the flux constant, line blanketed code MARCS (Gustafsson et

TABLE 2

DATA USED TO DERIVE $T_{\rm eff}$ AND log g FOR M13 STARS

M13 Star	$E(B-V) = 0.02$		$V_0 - M_V = 14.3$	
	V_0	$(B-V)_0$	$T_{\rm eff}$	log g
I-12	13.48	0.97	4600	1.70
I-65	13.99	0.83	4900	2.00
I-18	13.89	0.90	4750	1.90
II-28	13.56	0.88	4800	1.80
II-30	13.86	0.89	4750	1.90
IV-36	13.66	0.83	4900	1.90
IV-37	13.33	0.99	4600	1.60
IV-15	12.90	1.01	4600	1.40
IV-19	13.59	0.92	4600	1.80
I-49	13.93	0.87	4800	2.00

NOTE.—*UBV* photometry from Cathey 1974.

TABLE 3

ATOMIC DATA

Element	Wavelength (Å)	χ(eV)	log $gf\lambda$
Ca II	8498.02	1.69	2.49
Ca II	8542.09	1.70	3.28
Ca II	8662.14	1.69	3.17
Ca I	5581.97	2.52	3.19[a]
Ca I	5590.13	2.52	3.18[a]
Ca I	5594.47	2.52	3.84[a]
Ca I	5598.49	2.52	3.66[a]
Ca I	5601.29	2.53	3.23[a]
Ca I	5867.57	2.93	2.19[b]
Fe I	8387.78	2.18	2.43[c]
Fe I	8468.41	2.22	1.86[c]
Fe I	8611.81	2.85	1.89
Fe I	8621.62	2.95	1.81
Fe I	8688.64	2.18	2.73[c]
Na I	5682.63	2.10	3.04[d]
Na I	5688.20	2.10	3.35[d]
Na I	5889.95	0.00	3.88[d]
Na I	5895.92	0.00	3.58[d]

[a] Smith & Raggett 1981.
[b] Spite et al. 1987.
[c] Blackwell et al. 1982.
[d] Holweger 1972.

al. 1975). Synthetic spectra were generated from the models using the code SSG (Bell & Gustafsson 1978). These model atmospheres and resulting synthetic spectra were parameterized by temperature, gravity, metal abundance, and microturbulent velocity ($T_{\rm eff}$/log g/[A/H]/V_ξ).

Table 3 lists the lines and atomic data used in this analysis. In the case of the Ca triplet, some atomic data were determined by a simple fitting procedure. Two parameters had to be determined, the oscillator strength and the collisional broadening for the lines of the Ca triplet. These two parameters were determined by matching synthetic profiles to observed profiles of the Sun and Arcturus, with allowances being made for radiative damping. The models used were the Holweger-Müller (1974) solar model (using $V_\xi = 1.0$ km s^{-1}) and 4300/1.50/−0.5/1.7 MARCS model for Arcturus. The oscillator strength (f) and the van der Waals collisional broadening constant due to collisions with hydrogen atoms, C_6, were varied until the synthetic profiles fit the observed profiles. A Ca abundance of 6.36 on the logarithmic scale of $H = 12$ was used. The observed profiles are from the tape version of the Kurucz et al. (1984) solar atlas and the photometric atlas of Arcturus (Griffin & Griffin 1968). The results of this fitting technique compare very well with those found by a similar technique (Holweger 1972) and reasonably well with those from Gallagher (1967).

The oscillator strengths used for the other lines used in this analysis were taken from other sources (Smith & Raggett 1981; Blackwell et al. 1982; Holweger 1972; Spite et al. 1987). Their collisional broadening constants were determined using synthetic spectra of the Sun to match the equivalent widths given in Moore, Minnaert, & Houtgast (1968). The collisional broadening strengths were varied until the appropriate equivalent width was achieved. These results were then checked by requiring that the synthetic equivalent widths matched the observed widths given by Mäckle et al. (1975) for Arcturus. For most of the lines the collisional broadening had only a small effect on the total equivalent width. For example, even if we set $C_6 = 0$, the abundances derived from the Ca triplet would only increase by about 0.2 dex.

3.2. *Error Analysis*

The significant and estimatable sources of error are the gf-values, effective temperatures, gravities, and equivalent widths. We shall discuss each of these and by using the technique of varying the input quantities for the model 4600/1.70/−1.0/1.7 for star I-12 in M13, we shall try to estimate the magnitude of each source of uncertainty. The deviations found are considered representative of the whole set of stars.

An error in f is directly translatable into an error in the derived abundance. Since the oscillator strengths of the Ca triplet were determined by profile fitting, this analysis is sensitive to the choice of the Sun's and Arcturus's abundance as well as to the model used. As an example, Wallerstein et al. (1987) use a similar technique to determine f values, but they use a model which is 75 K hotter and has a gravity 0.2 dex higher for Arcturus. This results in a systematic difference of at least +0.15 dex in derived abundances between this analysis and theirs.

Uncertainties in $T_{\rm eff}$ and log g are also very important and reflect uncertainties in the distance moduli and color excesses. Crocker (1989), in her analysis of the reddening in the direction of M22, observes a scatter of about 0.05 magnitudes. If we interpret this scatter as being due entirely to differential reddening then an increase (decrease) of 0.05 magnitudes in the reddening would cause any model temperature and log g to be increased (decreased) by 100 K and 0.06 dex, respectively (from Bell & Gustafsson 1978). Changes of this magnitude were found to lead to changes of ± 0.03 and ± 0.06 dex for the abundance derived from the Ca triplet and ± 0.15 and ± 0.01 dex for abundances derived from the "weak" lines, respectively.

The results are also sensitive to errors in the measurement of equivalent widths. It was found that an error of 10% in the equivalent width, resulted in an error of about 0.1 dex in the final abundance. In this analysis, the error in equivalent width is probably entirely due to uncertainties in continuum location. This uncertainty could also lead to significant differences between this analysis and others.

Uncertainties in the microturbulence should be mentioned. For this analysis, a microturbulent velocity of 1.7 km s^{-1} was used. This figure seems fairly representative of what is known for red giants (Gustafsson, Kjaergaard, & Andersen 1974; Frisk 1984), but the actual value is unknown. If the high value of 2.7 km s^{-1} were used, the abundances found would decrease by 0.4 dex for Fe and the "weak" Ca lines, and about 0.02 dex for the Ca triplet.

There is also a difference in [Ca/H] as determined by the six weak lines and the Ca triplet. The [Ca/H] from the CaT is systematically 0.2 dex higher than the abundance derived from the six weak lines. This systematic difference is caused by the fact that when the log gf-values for the Ca triplet were determined, the line profiles were fitted in the wings. The cores of the lines (within about 0.2 Å of line center) were not fitted very well. In fact, the synthetic profiles were brighter in the cores than the observed for both the Sun and Arcturus. Thus, one would expect these log gf and model combinations to make the lines too weak and as a result need higher abundance to match the measured equivalent width.

Can we determine what causes the synthetic Ca triplet lines to be systematically too weak in the models? The effect most likely to be important in the core of strong lines is the temperature structure of the atmosphere at low optical depths. Ruland et al. (1980), in a study of Pollux, have found that the

TABLE 4

ESTIMATION OF UNCERTAINTIES

Uncertainty	Effect
$\Delta \log gf\lambda = \pm 0.1$	Δ[A/H] = ± 0.1
$\Delta E(B-V) = \pm 0.05$	$\Delta T_{\rm eff} = \pm 100$ K
	$\Delta \log g = \pm 0.1$
$\Delta T_{\rm eff} = \pm 100$ K	Δ[Ca/H]$_{\rm CaT} = \pm 0.03$
	Δ[A/H] = ± 0.15
	Δ[Na/H] = ± 0.10
$\Delta \log g = \pm 0.1$	Δ[Ca/H]$_{\rm CaT} = \pm 0.06$
	Δ[A/H] = ± 0.01
	Δ[Na/H] = ± 0.14
$\Delta W_\lambda = \pm 10\%$	Δ[A/H] = ± 0.1
$\Delta V_\xi = \pm 1$ km s^{-1}	Δ[Ca/H]$_{\rm CaT} = \pm 0.02$
	Δ[A/H] = ± 0.4
	Δ[Na/H] = ± 0.13
Quadrature sum	Δ[Ca/H] = ± 0.16
(without ΔV_ξ)	Δ[A/H] = ± 0.20
	Δ[Na/H] = ± 0.22

NOTE.—[A/H] represents the Ca and Fe weak line uncertainties.

Bell et al. (1976) models are too bright in the inner wings of the Ca triplet and other strong lines (as we also found). They suggest that this is caused by the theoretical models being about 200 K too hot at $\tau_{5000} = 0.1$. This result however was contradicted by Desikachary & Gray (1978) who, using a similar technique found the opposite tendency from a study of the K-line profile. Gustafsson (1983) suggests that these conflicting results may be due to the inherent difficulties in interpreting strong line profiles. However, if there are some problems with the models, it may be that real atmospheres have thermal inhomogeneities in their upper photospheres. Thus it seems that the strong line profiles are telling us of problems with the model atmospheres, but we can at present only guess what the problems are and how to solve them.

The bottom line of the above analysis is that the total random error in the results is probably 0.15 dex for all the lines used in this analysis (see Table 4 for a summary). This is not actually borne out by the scatter in the analysis of each star or in that of the whole group. For instance, the scatter in abundance for the Ca triplet from line to line is about 0.05 and for the Fe and "weak" Ca lines about 0.15. The star to star scatter is only 0.08 and 0.16 for the Ca triplet and 0.04 and 0.11 in the Fe lines for M13 and M22, respectively. Thus it seems that the actual uncertainties may be lower than that estimated from the above discussion.

4. RESULTS

The results of the analysis can be seen in the first five columns of Tables 5 and 6. As discussed in the previous section, these are probably accurate to about 0.2 dex. It is interesting to note that the scatter in abundance derived from the Ca triplet and the Fe lines is only 0.08 and 0.04 respectively for M13. For M22, these numbers are 0.16 and 0.11, about twice that of M13. Supposing that M13 is uniform in Fe and Ca abundance, this suggests that the internal uncertainty (ignoring systematic errors) is about 0.08 and 0.04 dex in the Ca triplet and Fe abundance analysis. If our supposition is correct, then it may be that the stellar population of M22 is not uniform in abundance or that there is some other effect increasing the scatter in only the M22 abundance determinations. One possibility that needs to be investigated in this latter case is the possibility of differential reddening along the line of sight.

In the spectra of M22 stars, strong interstellar Na D absorption lines are seen. They range in equivalent width from 0.91 Å and 0.75 Å (5889.95 Å and 5895.92 Å) for IV-20 to 0.64 Å and 0.46 Å for I-80. These are the two extremes and eight of the stars had Na D equivalent widths within 0.05 Å of the average (see Table 7). Our estimated uncertainty in our equivalent width measurements is about 10%, thus the similarity of equivalent widths is consistent with constant reddening for these eight stars. However, we should note that since the lines are also saturated and the amount and ionization degree of the interstellar gas is not directly linked to the reddening, the range of equivalent widths is also consistent with a variation in reddening of perhaps 30%–50%. Also, one of us (Cohen), has unpublished Thuan-Gunn g and i colors for two regions of M22. These two regions roughly correspond to quadrant I and quadrants III and IV. The main results of this analysis is that the color spread is larger than the errors, the field that contains the stars IV-20, IV-24, I-116, and I-53 is redder than the outer fields, the field that contains I-27 and I-108 seems to be bluer on average than the other fields, and that the total spread in reddening is about 0.1 magnitudes in $(g - i)$ color.

Crocker (1989) found a range in $E(B-V)$ of about 0.1 magnitudes with a mean of 0.42. If we adopt this range in color excess as being the range of reddening, what affect would that have? This would, as noted in the error analysis, result in an uncertainty of about ± 100 K and ± 0.1 dex in $T_{\rm eff}$ and log g.

TABLE 5

ABUNDANCES AND INDICES OF M22 STARS

Star	[Ca/H]$_{\rm CaT}$	[Ca/H]	[Fe/H]	[Na/H]	δA(Ca)[a]	δS(3839)[a]	W(G)[a]
I-116	−1.10	−1.21	−1.56	−1.38	−0.039	0.35	...
I-108	−1.09	−1.30	−1.41	−1.20	−0.035	0.01	...
I-53	−0.96	−1.04	−1.63	−1.58	0.039	0.62	12.3
III-25	−1.19	−1.30	−1.49	−1.79	−0.083	0.41	5.0
III-35	−1.13	−1.38	−1.72	−1.61	−0.038	0.11	6.2
IV-20	−0.76	−0.99	−1.51	−1.03	0.055	0.75	11.8
IV-24	−0.94	−1.02	−1.57	−1.40	0.022	1.05	11.7
I-80	−0.70	−1.03	−1.42	−1.33	0.031	0.92	11.0
I-85	−1.06	−1.38	−1.56	−1.40	−0.024	0.11	5.7
I-27	−0.82	−0.86	−1.48	−1.22	−0.078	0.24	9.0
Mean:	-0.99 ± 0.15	-1.15 ± 0.18	-1.54 ± 0.11	-1.43 ± 0.20			

[a] From Norris and Freeman 1983.

TABLE 6

ABUNDANCES AND INDICES OF M13 STARS

Star	$[Ca/H]_{CaT}$	[Ca/H]	[Fe/H]	[Na/H]	δm_{CN}[a]	m_{HK}[b]
I-12	−0.89	−1.06	−1.56	−1.26	0.228	0.440
I-65	−1.02	−0.96	−1.53	−1.25	0.158	...
I-18	−0.91	−1.01	−1.57	−1.38	0.253	0.401
II-28	−1.03	−1.09	−1.47	−1.54	−0.008	0.364
II-30	−0.78	−1.04	−1.50	−1.21	...	...
IV-36	−0.91	−0.91	−1.50	−1.52	−0.006	...
IV-37	−0.85	−1.04	−1.58	−1.27	...	...
IV-15	−0.95	−1.03	−1.55	−1.40	0.131	0.445
IV-19	−0.88	−1.06	−1.61	−1.89	...	...
I-49	−0.85	−0.99	−1.52	−1.50	0.152	0.418
Mean:	−0.90 ± 0.08	−1.02 ± 0.05	−1.54 ± 0.04	−1.42 ± 0.20		

[a] Suntzeff 1981.
[b] Suntzeff 1980.

An uncertainty of this magnitude would cause approximately ±0.1 dex in [Ca/H] derived from the Ca triplet and about ±0.15 uncertainty in the [Fe/H] analysis. However, the equivalent widths of the interstellar Na D lines in Table 7 may indicate that only IV-20 and I-80 have reddenings significantly different from the mean. Combining this with the fact that [Ca/H] from the Ca triplet is relatively insensitive to T_{eff} and to log g implies that differential reddening along the line of sight has little effect on narrowing the abundance spread in [Ca/H]. We do however caution the reader that until the reddening is determined spatially, our results should be viewed with some caution.

TABLE 7

EQUIVALENT WIDTHS OF THE INTERSTELLAR Na D LINES FROM THE M22 SPECTRA

Star	W_{5890} (mÅ)	W_{5896} (mÅ)
I-116	800	620
I-108	810	600
I-53	800	640
III-25	800	670
III-35	840	680
IV-20	910	750
IV-24	810	640
I-80	640	460
I-85	780	610
I-27	780	610
Mean:	800 ± 700	630 ± 70

How do these results compare with the results of other determinations? For M13, our results compare very well with the studies listed in Table 8. For M22 there are noticeable differences. These differences are almost entirely due to our choice of distance modulus and reddening. If we had chosen values of 12.4 and 0.36, instead of 12.20 and 0.42 for the distance modulus and the reddening, our mean for [Fe/H] and [Ca/H] would be about 0.2 dex lower. This would remove much of the disagreement between our mean abundances and those of other studies.

4.1. *Abundance Spread in M22*

The question of whether M22 really has a variation in abundance of metals other than C and N must be resolved. This

TABLE 8

COMPARISON WITH OTHER STUDIES

Study	[Fe/H]	[Ca/H]	Number of Stars	Variation (dex)	$(m-M)_0$	$E(B-V)$
			M22			
Norris & Freeman 1983	...	...	100	0.3[Ca/H]		
Peterson 1980a	−1.94	...	4	0.3[Fe/H]	12.45	0.36
Cohen 1981	−1.78	−1.07	3	No	12.45	0.36
Manduca & Bell 1978	−1.6	−1.25	6	No		
Gratton 1982	−1.94	−1.5	3	No	12.57	0.32
Wallerstein et al. 1987	−1.6	...	2	No	12.45	0.36
Pilachowski et al. 1982	−1.7	−1.4	6	0.5[Fe/H]	12.75	0.29
This study	−1.54	−1.15	10	0.5	12.2	0.42
Mean	−1.7	−1.3				
			M13			
Wallerstein et al. 1987	−1.6	...	2			
Peterson 1980b	−1.71	−1.41	3	[Na/H]		
Pilachowski et al. 1980	−1.4	−1.1	5			
Leep et al. 1980	−1.6	...	4			
Cohen 1978b	−1.6	−1.2	5	[Na/H]		
This study	−1.54	−1.02	10	0.5[Na/H]		
Mean	−1.6	−1.2				

study suggests that there is a small abundance spread, whereas most of the studies in listed Table 8 do not. To answer this question, we will look at the data of Norris & Freeman (1983). They carried out a low-resolution spectral investigation of 100 M22 stars in which they measured the following indices, defined by

$$S(3839) = -2.5 \log \int_{3846}^{3883} I_\lambda\, d\lambda \Big/ \int_{3883}^{3916} I_\lambda\, d\lambda$$

$$A(\mathrm{Ca}) = 1 - 0.956 \frac{\int_{3916}^{3985} I_\lambda\, d\lambda}{(\int_{3883}^{3916} I_\lambda\, d\lambda + \int_{3985}^{4018} I_\lambda\, d\lambda)}$$

$$W(\mathrm{G}) = \int_{4290}^{4318} (I_{4318} - I_\lambda) d\lambda / I_{4318}\, d\lambda\,,$$

where I_λ is the relative intensity determined from the spectra. $S(3839)$ measures the strength of the CN band at λ3883, A(Ca) measures the absorption in the region of Ca II H and K, and W(G) measures the CH absorption in the region of the G band.

A(Ca) measures the relative Ca II H and K line strengths (with a contribution from other lines including two Al resonance lines in the feature band). Their strength is not only a function of abundance, but also temperature and surface gravity. To correct for these dependencies Norris & Freeman did a linear least-squares fit to the data in their A(Ca) versus V_0 (reddening corrected visual magnitude) diagram. They defined δA(Ca) as the difference between the measured A(Ca) and their fit. Since δA(Ca) depends mostly on Ca abundance, we expect to see a correlation between our Ca abundances and δA(Ca). Figure 2 shows the strong correlation (correlation coefficient = 0.85) using our results from the six weak Ca lines. The Ca abundances derived from the Ca triplet also correlate about as strongly. The star I-27 was not included since its deviation in this diagram and others suggests that the star was misidentified in either the Norris & Freeman observations or in ours.

What does this strong correlation tell us? First, it confirms the idea that the abundance variation is real. If the abundance spread was due to errors and uncertainties in the analysis, we

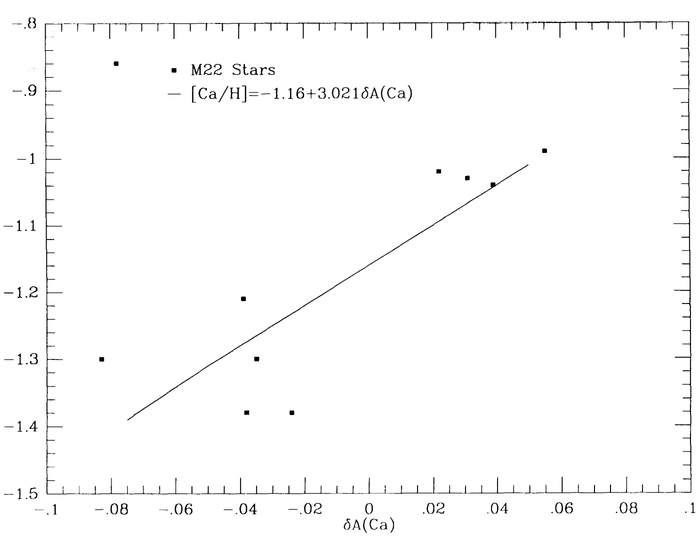

FIG. 2.—[Ca/H] from the six weak Ca lines versus the δA(Ca) index of Norris & Freeman (1983) for M22. The line is a linear least squares fit to the data (excluding the star I-27). The correlation between [Ca/H] and δA(Ca) is excellent.

TABLE 9

STAR BY STAR EXAMINATION OF THE RESULTS OF OTHER STUDIES

Star	[Fe/H]	[Ca/H]	Estimated[a]	δA(Ca)
Pilachowski et al. 1982				
III-3	−1.4	−1.3	−1.15	0.004
III-26	−1.6	−1.3	−1.25	−0.029
IV-97	−1.7	−1.5	−1.36	−0.066
IV-102	−1.8	−1.5	−1.24	−0.028
V5	−1.9	−1.7		
V8	−1.6	−1.4		
Cohen 1981				
II-31	−1.80	−1.10	−1.27	−0.036
IV-20	−1.66	−0.95	−1.05	0.035
IV-59	−1.87	−1.17	−1.20	−0.012
Gratton 1982				
I-92	−1.91	−1.6	−1.13	−0.011
III-12	−2.01	−1.6	−1.04	0.040
III-52	−1.89	−1.4	−1.08	0.027
Wallerstein et al. 1987				
II-3	−1.5	...	−1.16	0.004
IV-102	−1.7	...	−1.25	−0.028
Peterson (1980a)				
III-3	−1.62	...	−1.16	0.004
III-26	−1.81	...	−1.25	−0.029
IV-97	−2.15	...	−1.37	−0.066
V8	−2.18	...		

[a] From [Ca/H] = 3.02 δA(Ca) − 1.16.

would not expect to see such a strong correlation. The slope of the least-squares fit between [Ca/H] and δA(Ca), 3.02, is intermediate between values calculated by Norris & Freeman viz, $\partial\delta A(\mathrm{Ca})/\partial[\mathrm{Ca/H}] = 2.56$ and $\partial A(\mathrm{Ca})/\partial[A/\mathrm{H}] = 4.8$. Second, and most importantly, it allows us to transform δA(Ca) into [Ca/H] for the 100 stars in the Norris & Freeman data set and thus see if other studies should have seen an abundance variation. The results of this treatment of the Norris and Freeman data are shown in Table 9. It is apparent that the estimated [Ca/H] reproduces the observed variations quite well (although there are systematic differences) and shows that studies which did not find an abundance variation, should not have. Unfortunately, some of the stars used in analyses which found variations were not included in the Norris & Freeman study. Thus we are unable to confirm these variations. We can confirm, however, that studies which did not find abundance variations would not have been expected to do so. Our sample, on the other hand, was deliberately chosen to represent the extremes in A(Ca). It includes the star with the lowest measured A(Ca) as well as the star with the tenth highest.

Figure 3 shows the abundance histogram resulting from our treament of the Norris & Freeman (1983) data. It shows that there is a very small probability of seeing the stars at the extreme ends of the abundance range. This explains why the variation has been a point of controversy. Most studies examined an insufficient number of stars, over too limited of a range in A(Ca), to see the abundance variation.

A possible argument against this histogram representing the real abundance distribution is that it could be caused by errors and uncertainties. This is unlikely to be the case. First, the M13 results suggest that if the abundance variation was due to

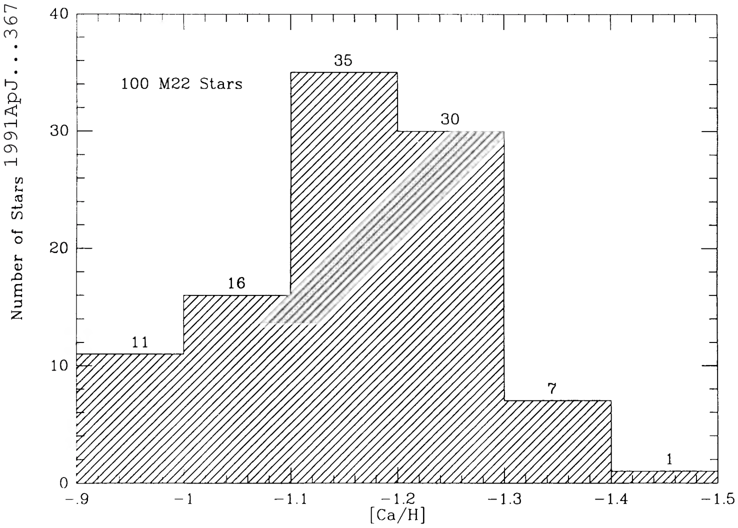

FIG. 3.—A histogram of the number of stars vs. [Ca/H] from the six weak Ca lines for M22. The histogram shows that the stars are not distributed randomly as a function of abundance, but is approximately Gaussian-shaped. This implies that a reasonable number of stars must be included in any sample to observe the abundance variation.

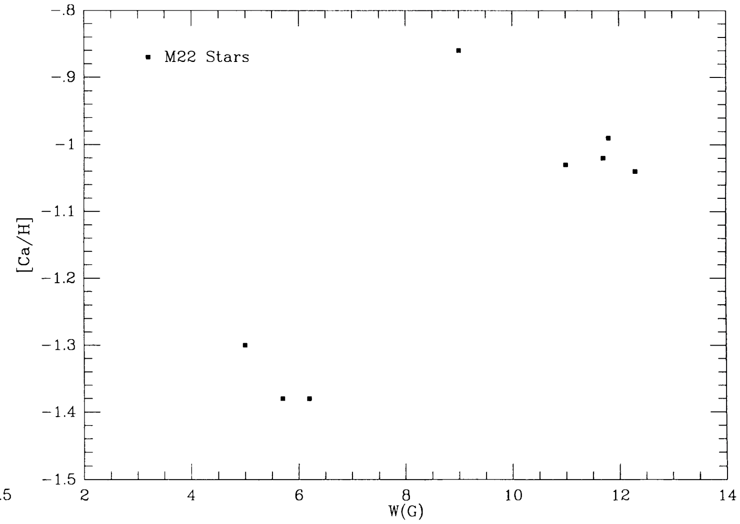

FIG. 4.—[Ca/H] from the six weak Ca lines vs. W(G) from Norris & Freeman (1983) for M22. The correlation is good, suggesting that the carbon abundance is correlated with the calcium abundance. This applies another constraint on the primordial hypothesis and weakens the argument for mixing as the cause of the variations.

errors in the observations and data analysis, then the distribution should be much narrower. Second, δA(Ca) is basically a measure of equivalent width, thus we expect the uncertainties in δA(Ca) to be small. The distribution of [Ca/H] shown in Figure 3 really represents the distribution of δA(Ca) since only a linear transformation was applied to go from δA(Ca) to [Ca/H]. If the variation of δA(Ca) is due to abundance variations in Ca, then this histogram represents the real distribution of abundance regardless of the scaling. Finally, the correlation between A(Ca) and our [Ca/H] is strong; the relationship has a correlation coefficient of 0.88 and a probability of determining such a correlation from an uncorrelated parent population of less than 0.001. Thus, it is highly unlikely that the correlation resulted from a completely uncorrelated parent population.

How about the correlation with the other bands strengths of Norris & Freeman? Figures 4 and 5 show the correlations of [Ca/H] with W(G) and δS(3839). The correlations are strong, with correlation coefficients of 0.80 and 0.91 (omitting I-27), respectively. W(G) measures the strength of CH absorption [higher W(G) implies stronger G band absorption], which is proportional to carbon abundance. δS(3839) is defined as the height a star falls above the baseline on a diagram of S(3839) versus V. The baseline is given by $S(3839) = -0.090V + 1.065$. The baseline attempts to correct for the variation of temperature and gravity along the red giant branch. Thus δS(3839) represents a measure of CN strength irrespective of luminosity. Norris & Freeman, assuming $[O/A] = 0.0$, derive carbon abundances for three giants used in this study. We find that [Ca/H] correlate with their derived $[C/A]$ for these three stars.

These correlations could, however, imply another effect other than abundance variations. It has been suggested that CN and CO enhancements could lead to structural changes (cooling) in the upper atmosphere of red giants, causing enhancement of low-excitation potential lines (Peterson 1976). Ca has a low excitation potential and the Ca triplet is formed in the upper atmosphere, so it might suffer from the effects of increased CN and CO cooling. First M13 was found to be uniform in Ca abundance even though there are variations in CN line strengths. Second, Campbell & Smith (1987) looked at CN-rich stars in NGC 6752, M4 and 47 Tucanae, with Na I and Al I line enhancements, to determine if these enhancements are accompanied by enhancements of the λ7699 K I line, a line with a low excitation potential. If there was a positive correlation between Na, Al, and K line strengths then atmospheric cooling might be the cause of the enhancement. They found no such correlation, the K line strengths were uniform in CN-poor and CN-rich giants.

M22 is found also to vary in Na abundance (see Fig. 6). The range of [Na/H] seems to be about 0.8 dex and correlates reasonably well with the calcium abundances found from the Ca triplet and W(G) (correlation coefficient 0.64; see Fig. 6). Sodium, however, does not seem to correlate with [Fe/H] or

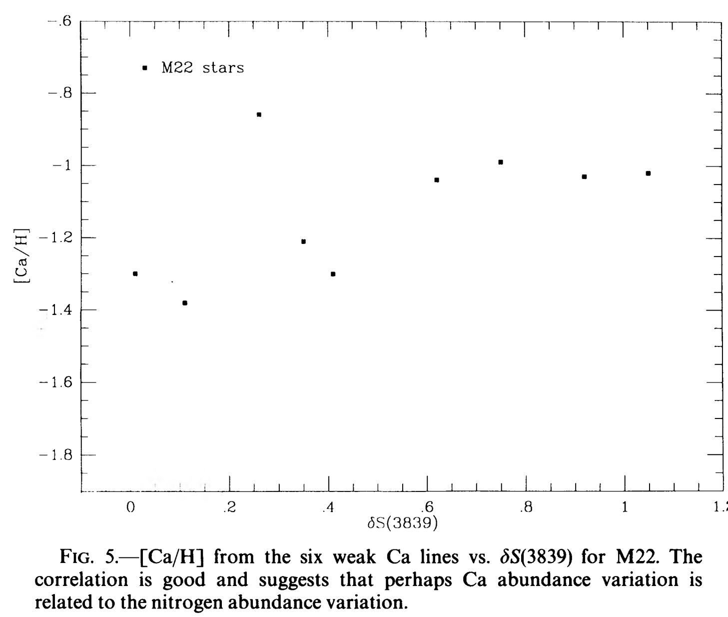

FIG. 5.—[Ca/H] from the six weak Ca lines vs. δS(3839) for M22. The correlation is good and suggests that perhaps Ca abundance variation is related to the nitrogen abundance variation.

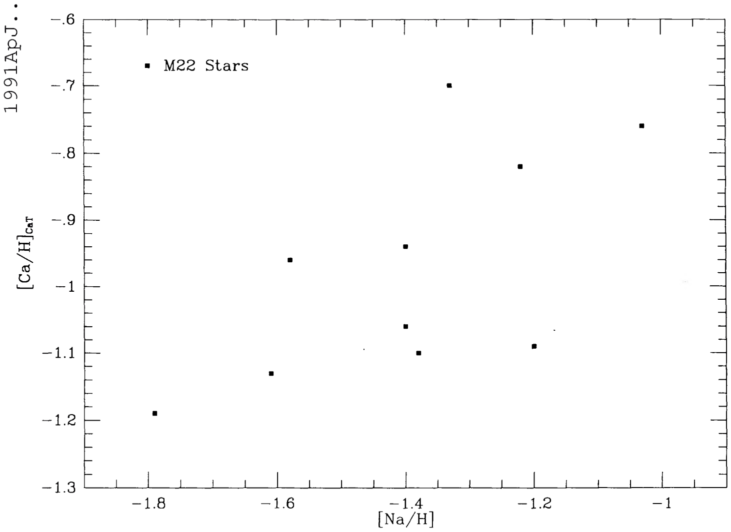

FIG. 6.—[Ca/H] from the Ca triplet vs. Na abundance for M22. The correlation between sodium and calcium is reasonable.

$\delta S(3839)$. The lack of correlation with $\delta S(3839)$ and the weakness of the other correlations is surprising but not totally unexpected. The line to line scatter in the sodium abundances for individual stars was much higher than for either the Fe or Ca analyses. As can be seen from the estimation of uncertainties given in Table 5, we expect [Na/H] to be more uncertain than either [Ca/H] or [Fe/H].

The correlations between [Fe/H] and the other line strengths and abundances are not as strong. In Figure 7 we show that [Ca/H] is at most weakly correlated with [Fe/H] (correlation coefficient = 0.37), so one would expect correlations between Fe abundance and the other line strengths and abundances. This is indeed the case, but the correlations are reduced. This may be due to uncertainties in the analysis to which the weak lines are preferentially more sensitive, such as temperature calibration and the chosen value of the microturbulence. The Ca triplet is relatively insensitive to temperature and microturbulence (see Table 5). However, this suspicion is not verified by the results for the "weak" line Ca abundances. If our suspicion was correct, we would expect the correlations found using the Ca triplet to be much stronger than those using the "weak" Ca lines. What we find is that the correlations using the Ca triplet and the "weak" Ca lines are about as strong. It is possible that there is inherently more scatter in the Fe abundances than in the Ca abundances, or perhaps the source of the Fe variations is different than that of the Ca abundances.

4.2. *Abundance of M13*

What about M13? Are there any correlations? Suntzeff (1980) and (1981) tabulates the following measures for some M13 stars:

$$m_{\rm HK} = -2.5 \log \frac{\int_{3910}^{4020} F_\lambda \, d\lambda}{\int_{4020}^{4130} F_\lambda \, d\lambda}$$

$$m_{\rm CN} = -2.5\left(\log \int_{3850}^{3878} F_\lambda \, d\lambda/28\ \text{Å} - \log \int_{3896}^{3912} F_\lambda \, d\lambda/26\ \text{Å}\right).$$

We attempt to compensate for variations in temperature and gravity by defining $\delta m_{\rm HK}$ and $\delta m_{\rm CN}$. $\delta m_{\rm HK}$ is the distance from the reference line $0.495(B-V)_0 - 0.041$, which is a linear fit to the $m_{\rm HK}$ data in Suntzeff (1980) in the plot of $m_{\rm HK}$ versus $(B-V)_0$. Similarly, $\delta m_{\rm CN}$ is the distance from $0.341(B-V)_0 - 0.227$, which is a linear fit to the CN-poor stars in Suntzeff (1981). As expected, even with just five points, [Ca/H] is correlated with $\delta m_{\rm HK}$ and [Fe/H] and [Ca/H] are not correlated with $\delta m_{\rm CN}$ although the range in CN strength is large (about $\frac{3}{4}$ of the stars are classified as CN strong, approximately $\delta m_{\rm CN} >$ 0.1, the rest are CN weak). It should be noted that Suntzeff lists his dispersion in $m_{\rm HK}$ as 0.030, about half the overall range. Suntzeff states that this is consistent with an abundance variation of about 0.07 dex if the Ca abundance is coupled with the Fe abundance. This is indeed about what we see.

The Na variation in M13 was first found by Cohen (1978b), who reported finding a variation of about 0.6 dex. This compares very well with the present values. There is some correlation (correlation coefficient = 0.73) between [Na/H] and $\delta m_{\rm CN}$ (see Fig. 8). Suntzeff (1981) also derived N and C abun-

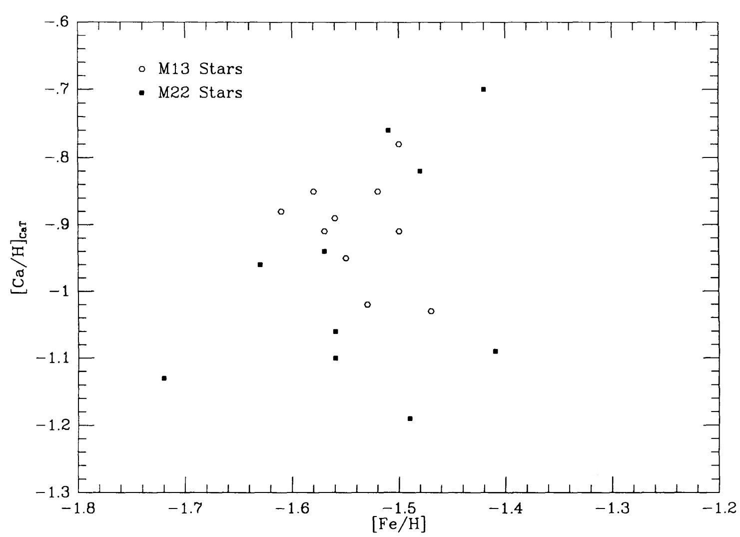

FIG. 7.—[Ca/H] from the Ca triplet vs. [Fe/H] from the five infrared iron lines. The plot represents a correlation between Ca and Fe abundances for M22 and the relative uniformity of the abundances in M13. The correlation between Fe and Ca abundances for M22 is weak to nonexistent.

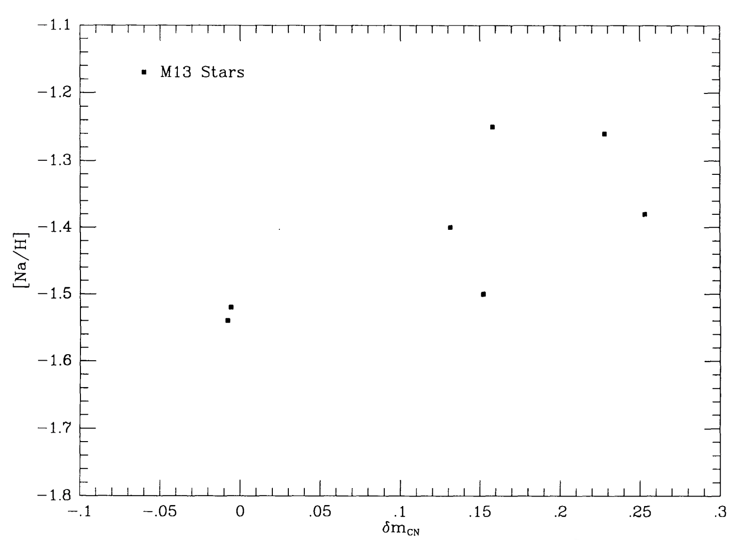

FIG. 8.—Sodium abundance vs. $\delta m_{\rm CN}$ from Suntzeff (1981). The correlation is good. This suggests that the source of the Na variation may be the same as that of the N variation.

dances and our determinations of [Na/H] correlate with his [N/Fe], but not with [C/Fe]. It should be kept in mind that he claims a rough anticorrelation between [C/Fe] and [N/Fe] for M13. Over our sample we see no such anticorrelation in his results.

5. DISCUSSION

5.1. *The Case for a "Primordial" Cause for the Variations in M22*

If a globular cluster exhibits variations in elemental abundances, the key to determining the cause of the variations lie in how these variations correlate with one another. In M22 we have seen that [Ca/H] is correlated with [Na/H], $\delta S(3839)$, $\delta A(\mathrm{Ca})$, and $W(\mathrm{G})$. However, the correlations are not of all the same strength. [Ca/H] correlates very well with $\delta S(3839)$, $\delta A(\mathrm{Ca})$, and $W(\mathrm{G})$. There is some correlation between sodium abundance and calcium abundance, and at most a weak correlation between iron and calcium abundances. This may imply that the source of the Ca variation is the same as that for the CN and CH variations. For sodium the case is less clear. The weaker correlation between [Na/H] and [Ca/H] is probably due to uncertainties in the analysis. The reduced correlations of [Fe/H] may imply that the source of the iron variation is different than that between the other elements studied here.

Another important correlation in light of the trends found above is that of $\delta S(3839)$ and $W(\mathrm{G})$ found by Norris & Freeman. This is in contrast to the anticorrelation that is commonly found in cluster giants (see Smith 1986 for review). This correlation implies that C and, possibly, N are enhanced in Ca-rich stars. Our correlations suggest that the calcium enhancement is accompanied by C and, possibly, N enhancements. Since Ca is not thought to be produced in the interiors of these low-mass red giants, this implies that the cause of the variations may be "primordial."

Possible causes and mixtures of stellar masses that lead to an abundances variation of the type exhibited by M22 will not be given here. The reader is referred to the discussions and the references therein by Norris, Freeman, & DaCosta (1984), Smith (1986), Smith & Norris (1982), Norris & Smith (1983), and Norris & Pilachowski (1985). However, we would like to stress the importance of the weak correlations between [Fe/H] and the other variations discussed earlier. Scatter and uncertainty in our abundances may not be enough to remove the lack of correlations, this effect is most likely real. Heretofore, this lack of correlation has not been discussed. Wheeler, Sneden, & Truran (1989) discuss possible contributors of Fe, as well as those of other elements. They are of the opinion that Type Ia supernova produce much of the iron-peak nuclei. If this is the case, perhaps the lack of a strong correlation is telling us that the C, N, Na, and Ca were not significantly produced in this type of supernova. Certainly, our results seem to indicate that for M22 the site of Fe production may have been different than that of the other elements studied here. However, only a more detailed abundance analysis considering *r* and *s* process elements as well as oxygen will settle the question of where the sites of nucleosynthesis were contained.

5.2. *Sodium Enhancements in M13*

M13 seems to exhibit a slightly different phenomenon. While not having any variations in Ca and Fe abundances, it does show Na enhancements that are reasonably well correlated with CN enhancements. We have found through the use of C and N abundances from Suntzeff (1981) that [Na/H] seems to be correlated with [N/Fe] but not with [C/Fe]. This suggests that the Na enhancements may be coupled to the N enhancements.

If the Na enhancements are coupled to N enhancements, what could be the possible explanations? If non-LTE effects can be ruled out, then as suggested by Suntzeff (1989) three conclusions seem possible:

1. Na is created as a result of deep mixing and CN processing while the star is on the red giant branch;
2. the Na enhancements cause mixing to be more efficient;
3. the stars are primordially enhanced in both N and Na and that mixing occurs along the red giant branch creating the C-N anticorrelation.

The final point seems most tenable. There is mounting evidence for CN band strength variations on the subgiant branch and near the main-sequence turn-off of globular clusters (e.g., Bell, Hesser, & Cannon 1983). This is at a lower luminosity than most theoretical studies have indicated that mixing should occur. Thus it seems that mixing alone may be insufficient to explain every abundance variation.

Our data suggest that sodium is correlated with nitrogen, this being the case perhaps the C-N anticorrelation is the result of mixing adjusting a primordial enhancement of nitrogen. Thus in this scenario, the cluster begins with an excess nitrogen abundance and mixing adds to this excess, resulting in an C-N anticorrelation. This picture fits the available data fairly well.

6. CONCLUSIONS

1. M13 contains a RGB of uniform abundance with $[\mathrm{Ca/H}] = -0.95 \pm 0.15$ and $[\mathrm{Fe/H}] = -1.54 \pm 0.15$. The variations in CN strengths are not correlated with [Ca/H] or [Fe/H] (the Ca and Fe abundances are constant while the CN strengths vary from star to star).

2. Stars in M13 have different [Na/H] values. This variation correlates with CN band strengths suggesting that the source of Na variations may be the same as that of N.

3. M22 stars vary in [Ca/H], [Na/H], and [Fe/H] and variations in the former two correlate with Ca II H and K, CH, and CN strengths (CN and CH band strengths correlate). Since these variations correlate the source of the variation must be something other than mixing of CNO–processed material. However, the tightness of the correlations are not uniform. The {[Ca/H]–$\delta A(\mathrm{Ca})$}, {[Ca/H]–$W(\mathrm{G})$}, and {[Ca/H]–$\delta S(3839)$} relations seem tighter than the {[Ca/H]–[Fe/H]} relation. This implies that if the abundance variations are primordial, there may be a different source for the iron variations compared to that for Ca, C, and N.

4. The histogram of abundances in M22 indicates that the variation has a standard deviation of about 0.2 dex. Since this standard deviation is close to the uncertainty in most studies, it suggests that a study of a random sample of a few stars (<5) has a small probability of observing an abundance variation. So it is not surprising that most studies undertaken to data do not reveal the variation.

5. M13 and M22 are two different types of clusters. M22 is ω Cen–like; that is, it displays abundance variations with correlations between Ca abundances and CN and CH line strengths. M13 is a M92–like cluster in that it has a narrow RGB in the color-magnitude diagram, has no variations in Fe or Ca, while CN shows a nonbimodal distribution. This suggests that one hypothesis, primordial or mixing, alone may not

be enough to explain the richness of phenomena observed in globular clusters. M22 displays clear evidence for primordial enrichment although mixing must also be occurring, since evidence for it is observed in every globular cluster, but the relative contribution of mixing to the total variation must be small.

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